A Multi-Sector Kaleckian Model of Growth, Distribution and Structural Change along Pasinettian Lines

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Abstract

With this inquiry, we develop a disaggregated version of the Pos-Kaleckian growth model, by building connections of this framework with the Pasinettian structural economic dynamics approach. By relying upon vertical integration, it becomes possible to carry out the analysis initiated by Kalecki and followed by Dutt (1984) and Rowthorn (1982) within a multi-sectoral model. By using this approach, we consider the implications of income distribution on structural change and growth, with the structural economic dynamics being conditioned not only to patterns of evolving demand and diffusion of technological progress but also to the distributive features of the economy. By taking a step further, it is also possible to extend the analysis to the Neo-Kaleckian approach advanced by Bhaduri and Marglin (1990) to a multi-sectoral analysis. In this vein, it emerges the possibility that each sector in the economy exhibits either a profit-led or a wage-led regime, being the profit share, or the wage share, the policy variable that can be used to stimulate growth within sectors.

Keywords: growth and distribution models, structural change, investment allocation, multisectoral models.

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1. Introduction

The pos-Kaleckian [Kalecki (1954, 1968)] and Pasinettian [Pasinetti (1981, 1993)] growth models share some common principles. The relevance of the principle of effective demand in the short and long run (as opposed to the scarcity principle), the conception of capital as a commodity and vertical integration between sectors are just some common characteristics of both approaches[1]. Although sharing the Cambridge heritage, these models belong to different strands of the literature. The Pasinettian model is neo-Ricardian in essence with strong connections with the Sraffian framework, while the pos-Kaleckian framework has broad influences of the seminal works of Kalecki and Steindl. While the latter focuses on distributive issues and their effects on short-term positions, Pasinetti, on the other hand, delivers a long-run analysis of structural change.

The Pasinettian or Structural Economic Dynamic – SED hereafter – approach is distinguishable by its simultaneous considerations of supply and demand in a disaggregated framework, whereby the interaction between the evolving patterns of demand and technological progress is responsible for particular dynamics of output, prices and structural transformation of economies in different stages of the development process. The outcome of the Pasinettian analysis is mostly a normative approach, which highlights the equilibrium dynamic path of the economy and the difficulties to keep it when the economy departs from an equilibrium position.

Notwithstanding the relevance of the Pasinettian contribution, some authors have pointed to the necessity of a more positive approach to the SED framework. That would require, for instance, to carry out the analysis regarding the actual path of economic

[1] Focusing on a reconciliation between the Kaleckian effective demand and Sraffian normal prices, Lavoie (2003, p. 53), for instance considers that "a large range of agreement has remained, in particular about a most crucial issue, the causal role played by effective demand in the theory of capital accumulation".

variables within a multi-sectoral economy. But to deliver that, it is necessary to introduce some behavioral hypothesis of the economy. It is not possible, for instance, to study capital accumulation without some assumptions on what variables affect investment decisions, and on what agents will make such decisions. Insofar as the Pasinettian analysis is carried out at a pre-institutional level, this is precisely the point where the Kaleckian analysis can contribute to SED analysis. The pos and neo-Kaleckian literature made explicit assumptions on the investment functions and, by considering that savings and investment should equal *ex-post*, such approaches allow us to determine the growth rate of capital accumulation, with the profit rate being the adjusting variable.

But the Kaleckian literature that unfolded [see Dutt (1984) and Rowthorn (1982) and Bhaduri and Marglin (1990)] has mainly focused on distributive issues, with limited scope for disaggregated topics. Exceptions are the works by Dutt (1988), Park (1995), Lavoie and Ramírez-Gastón (1997) and HoonKim and Lavoie (2017). These authors have delivered two-sector models in the Kaleckian tradition that in principle could be adopted to study structural change. But the focus of these approaches is mainly on issues such as profit equalization, convergence between the actual and the normal rates of capacity utilization and overdetermination. In fact, such frameworks exclude the possibility of structural change in the long run since the steady state position of the model requires that the two sectors grow at the same rate.

That is somewhat surprising insofar as Kalecki (1954, 1968) himself had considered an economy with three compartments or sectors, and his digression on mark-ups relies implicitly on a multisectoral approach that focuses on the comparison of the sectoral with the average mark-ups. In fact, a great deal of the Pos-Keynesian literature uses verbal reasoning to approach disaggregated issues, but few arguments unfold regarding formal models. In many of his writings, Kaldor (1966, 1970) for instance made it clear that it was impossible to understand the growth process without a sectoral outlook, which distinguishes between increasing returns activities on the one hand and diminishing returns activities on the other. Notwithstanding the importance assigned by Kalecki and Kaldor to a disaggregated analysis, the Keynesian demand-oriented theories of output growth have not thoroughly and systematically incorporated structural change, the main reason why being the fact that the models in this tradition, like the models in the mainstream, consider national economies in the aggregate.

In the present paper, we show that the cross-fertilization between the Kaleckian and Pasinettian models may yield new results related to income distribution and structural change. The standpoint of the analysis is the concept of vertical integration which allows us to establish a correspondence between the two approaches[2]. In our view, this connection is essential insofar as an uneven distribution of income may affect the level of saturation of goods. The demand for a specific product may saturate for one of the social classes, while it can be expanding for another class, with lower per capita income. To the best of our knowledge, Araujo and Teixeira (2015) and Nishi (2014) were one of the few authors that considered the role that particular income distribution might play in generating different patterns of structural change.

In the present analysis, we aim at going a step further in the study presented by Araujo and Teixeira (2015). While such authors have connected the Pasinettian model with the Pos and Neo-Kaleckian one sector models, here we make this connection with the two-sector version of the Kaleckian model presented by Dutt (1988). The two-sector Pos-Kaleckian model allows tackling issues such as investment allocation, profit

[2] Scazzieri (1990, p.26) shows that "[a]ny given economic system may generally be partitioned into a number of distinct subsystems, which may be identified according to a variety of criteria. These are subsets of economic relationships that may be identified by the logical device of *vertical integration* (...)".

equalization, etc., which are intrinsic to a multisectoral analysis. By showing how to connect this model with the SED, the approach allows us to unfold an analysis of structural change and income distribution simultaneously, with new insights for both streams.

Our strategy consists in showing that the equations of the two-sector Pos-Kaleckian model and a two-sector version of the Passinetian multisectoral model are similar with the proviso that Pasinetti does not consider institutions. That means, for instance, that a pure labour theory determines the price level, and not mark-up pricing rationale. But once we assume a particular institutional set-up for the Pasinettian economy and introduce investment functions, we show that both models are equivalent, with the level of disaggregation being the only difference between them. To carry out our analysis, we will make use of a third model in the post-Keynesian tradition, namely the Feldman's model (1929), which allows us studying the interaction between distributive features and investment allocation.

We organize this paper as follows. In addition to this introduction, the second section presents on the Pos-Kaleckian growth, focusing on a controversy between Park and Dutt. In the third section, we use the Feldman two-sector model to offer a possible solution to the dispute and to build a bridge between the model and the Pasinettian model. After discussing the Pasinetti model, the fourth section highlights the similarities amongst the equations of the two models. Also, it is presented the conditions to pass from one model to the other, with a particular role played by Feldman's model. Finally, the last section summarizes the main conclusions.

2. The Pos-Kaleckian model of growth and distribution

An essential characteristic of the Kaleckian literature is the existence of independent investment and savings functions that depends on income distribution [Lavoie (1992)]. The saving propensities, for instance, are particular to each class may it be workers or capitalists. The rationale is that investment is determined mainly by the availability of credit in the financial sector as well as the 'animal spirits.' Once the investment is made effective, the demand determines output which in turns determines savings. Another essential feature of the model is that it adopts the device of vertical integration to obtain a multi-sectoral framework. Kalecki (1983, 1954) has assumed vertically integrated industries in the sense that they do not buy inputs from each other [see, e.g., Lopes and Assus (2010)], so that, the total value of production can be broken down into wages plus gross profit. Bhaduri and Marglin (1990, p.377) supported this view, by considering that in the pos-Kaleckian model "we can think of the representative firm as vertically integrated using, directly and indirectly, a constant amount of labour per unit of final output."

One of the advantages of this device is that allows us focusing on different levels of aggregation for the economic system. For the Pasinettian model, for instance, it was adopted to yield an economy with an arbitrary number of sectors while the Kaleckian literature unfolds regarding one or two sector models. Hence, as the starting point of the present analysis, we can disaggregate the Neo and Pos-Kaleckian models into an arbitrary number of vertically integrated sectors. Here we focus on the two-sector model insofar as we can present the Pasinettian prototype model regarding a bi-sectoral model, but the analysis extends easily to an arbitrary number of sectors. The Pos-Kaleckian model has two vertically integrated productive sectors and considers the following hypothesis.

Sectors 1 produces consumer goods while sector $k1$ produces capital goods. Each sector uses labor and capital from fixed coefficients of production with constant returns of scale. Besides, there are two social classes, the wage-earning workers who spend their entire income on the consumer, the profit-making capitalists who save them all. Workers income is the same in both sectors. From these hypotheses, Park (1995) presents the following equations:

$$p_1 = p_{k1} r \left(\frac{v_1}{u_1}\right) + W l_1 \tag{1}$$

$$p_{k1} = p_{k1} r \left(\frac{v_{k1}}{u_{k1}}\right) + W l_{k1} \tag{2}$$

$$1 = w(l_1 + l_{k1} x) \tag{3}$$

$$x = g_1 \left(\frac{v_1}{u_1}\right) + g_{k1} \left(\frac{v_{k1} x}{u_{k1}}\right) \tag{4}$$

$$W = w p_1 \tag{5}$$

$$p_1 = (1 + \theta_1) W l_1 \tag{6}$$

$$p_{k1} = (1 + \theta_{k1}) W l_{k1} \tag{7}$$

$$g_1 = g_1(r_1, u_1) \tag{8}$$

$$g_{k1} = g_{k1}(r_{k1}, u_{k1}) \tag{9}$$

$$r_1 = r \tag{10}$$

$$r_{k1} = r \tag{11}$$

$$g_1 = g \tag{12}$$

$$g_{k1} = g \tag{13}$$

Where x stands for the output of the investment good relative to the output of the consumption good; p_1 and p_{k1} represents the price of the product of sectors 1 and $k1$, respectively; r_1 and r_{k1} is the rate of profit in sectors 1 and $k1$, respectively; v_1 and v_{k1} consist of the capital-full capacity output ratio in sectors 1 and k1 respectively; u_1 and u_{k1} consist of capacity utilization in sectors 1 and $k1$ respectively; W is the nominal wage rate; w the real wage rate; l_1 and l_{k1} are the labour-output ratio in sectors 1 and $k1$ respectively; x is the ratio of the sector $k1$ product to the sector 1 product; g_1 and g_{k1} are the sectoral accumulation rates for of sectors 1 and $k1$, respectively, and θ_1 and θ_{k1} are the mark-up for sectors 1 and $k1$, respectively.

Equations (1) and (2) are the sectoral decomposition of the product between wages and profits, while equation (3) shows that the output of the consumer goods sector corresponds to the sum of the workers' costs of the two sectors. Equation (4) indicates that the capital goods corresponds to the amount of the investments of both sectors while equation (5) is the real wage rate. Equations (6) and (7) are the mark-up prices, and equations (8) and (9) are the accumulation equations for both sectors. Finally, expressions (10) to (13) show that profit rates and accumulation rates between sectors tend to become uniform[3].

There are 13 equations and 12 unknowns (p_1, p_{k1}, x, u_1, u_{k1}, g, r, r_1, r_{k1}, g_1, g_{k1}, w), which over determinates the system [see Park (1995)]. To overcome the issue of over determination Park (1995) considered the possible solutions: a) to eliminate the uniform profits rate; b) to eliminate the uniform accumulation rate; or c) to eliminate the

[3] From (10) and (11) we obtain $r_1 = r_{k1}$ and from (12) and (13) we obtain $g_1 = g_{k1}$. However, the equations remain as they are to be faithful to Park's (1995, p. 300) exposition: "Dutt (1990) argues that function [2.8 and 2.9] should properly be thought of as a reduced-form equation, which shows how investment plans are made in equilibrium, rather than as a behavioral equation"

mark-up prices differences. But, after a detailed analysis of each of these possibilities, he has concluded that none of them make sense within a Kaleckian two-sector[4] model. That view was not accepted by Dutt (1997) who claimed that there is a misinterpretation in Park's (1995) argument insofar expressions (8) and (9) should be replaced by a "reduced-form equation, which shows how investment plans are made in equilibrium, rather than as a behavioural equation" [Dutt (1990, p. 117)]. Accordingly, the decisions on investment should be summarized by only one investment function that depends on the uniform profit rate and capacity utilization rates in each of the sectors, namely:

$$g = g(r, u_1, u_{k1}) \tag{14}$$

With this approach, the problem of overdetermination is eliminated insofar as expressions (8) and (9) are replaced by expression (14), yielding a system with twelve equations and twelve unknowns. However, despite the relevance of Dutt's view, Park's challenge based on the use of an aggregate accumulation function as an inadequate representation of investment decision in a multi-sectoral model remains. If expression (14) is not 'behavioural' as claimed by Dutt, it could not correctly represent the Keynesian autonomy of investment, and we should replace it by two disaggregated accumulation functions, namely (8) and (9). In the next section, we show that by using the Feldman two-sector model it is possible to show that model is precisely determined even in the presence of two disaggregated investment functions thus concluding that each author is

[4] Park (1995) criticizes Dutt's (1990) proposal to adopt the second solution, which is based on the view that investment should not be considered a behavioral variable. He also criticizes the two other solutions to solve the model overdetermination. In case of different profit rates, he recognizes the possibility of short-run divergence but asserts that it will hardly maintain in the long term, and on the alternative of mark-up pricing, Park emphasizes that, if the accumulation rate varies in different ways in different sectors, consequently the mark-up is changed proportionally.

right in his own terms. From algebraic manipulations of expressions (1) to (5)[5], it is also possible to show after some algebraic manipulation that:

$$r = g \tag{15}$$

Note that expression (15) is nothing but the Cambridge equation under the hypothesis that capitalists save all their income. It is also possible to obtain the following relations between the profit rate and rate of utilization capacity for each of the sectors [Dutt (1997)]:

$$r_1 = \frac{\theta_1}{1+\theta_{k1}}\frac{l_1}{l_{k1}}\frac{u_1}{v_1} \tag{16}$$

$$r_{k1} = \frac{\theta_{k1}}{1+\theta_{k1}}\frac{u_{k1}}{v_{k1}} \tag{17}$$

Expressions (16) and (17) are the profit-cost curve for the consumption and capital goods sector respectively. From (3) and (5), we conclude the level of the output of the investment good relative to the output of the consumption good is given by:

$$x = \theta_1\frac{l_1}{l_{k1}} \tag{18}$$

To obtain a further characterization of the long-run solution of the model, it is necessary to specify the accumulation function g, since it determines the growth rate of accumulation and consequently determines the long-run steady state. In the next section, we adopt particular investment functions and show that it is possible to obtain a closed form solution for the model by introducing a new variable, which is the rate of investment allocation. This variable arises from the Feldman's two-sector growth model and we show that it is the missing variable in the system insofar that by considering it the number of variables equals the number of equations of the model.

[5] In order to obtain this result, we isolate the capital-product relations $\frac{v_1}{u_1}$ and $\frac{v_{k1}}{u_{k1}}$ from (1) and (2), and insert them into equation (4). Similarly, from (3) and (5) we obtain that $p_1 = W(l_1 + l_{k1}x)$, and replacing it in (4) again and we obtain $r = g$ after some algebraic manipulation.

3. An Alternative Approach to the Kaleckian Model by using the Feldman model of Investment Allocation

In what follows, we show that the analysis developed by Feldman (1928) may yield an alternative approach to the Pos-Kaleckian growth model, which takes into account the decisions of investment allocation on economic growth as indicated by Araujo and Teixeira (2012). By adopting this approach, we determine the rate of investment allocation according to the equilibrium decisions of investment and savings as in the Kaleckian view. In what follows we adopt an alternative version of the Feldman two-sector growth model (1928) that takes into account the possibility of under-capacity utilization. The structure of the model is the same of the pos-Kaleckian model presented in the previous section, but now we make explicit the production function for sectors 1 and $k1$:

$$X_1 = \frac{u_1 K_1}{v_1} \tag{19}$$

$$X_{k1} = \frac{u_{k1} K_{k1}}{v_{k1}} \tag{20}$$

With this assumption and considering that $K = K_1 + K_{k1}$ is the total capital stock, it is possible to show that expression (4) is equivalent to: $\dot{K} = I = X_{k1}$. Such expression shows that in the absence of depreciation, the aggregate investment, namely I, is equal to the output of investment good sector. We assume that a proportion λ of the current production of the investment sector is allocated to itself while the remaining, $1 - \lambda$, is allocated to sector 1 ($0 \leq \lambda \leq 1$), which yields:

$$\dot{K}_1 = (1 - \lambda) X_{k1} \tag{21}$$

$$\dot{K}_{k1} = \lambda X_{k1} \tag{22}$$

By inserting expressions (19) and (20) into (21) and (22), it yields after some algebraic manipulation, the growth rates of sectors 1 and $k1$ from the supply viewpoint:

$$\frac{\dot{K}_1}{K_1} = g_1 = \frac{(1-\lambda)xu_{k1}}{v_{k1}} \tag{23}$$

$$\frac{\dot{K}_{k1}}{K_{k1}} = g_{k1} = \frac{\lambda u_{k1}}{v_{k1}} \tag{24}$$

In what follows, let us assume particular investment functions for each of the sectors. By adopting this specification, we agree with Park (1995) who considers that an aggregated investment function not being 'behavioral' cannot rightly be regarded as representing the Keynesian autonomy of investment function. To obtain a closed-form solution for the model, let us follow Kim and Lavoie (2017) who assume linear investment functions for sectors 1 and $k1$, respectively:

$$g_1 = \frac{I_1}{K_1} = \theta_1 + \alpha_1 r_1 + \beta_1 u_1 \tag{25}$$

$$g_{k1} = \frac{I_{k1}}{K_{k1}} = \theta_{k1} + \alpha_{k1} r_{k1} + \beta_{k1} u_{k1} \tag{26}$$

Where θ_1 and θ_{k1} denote autonomous growth rate of investment conveying the idea of animal spirits for sectors 1 and $k1$, respectively. r_1 and r_{k1} are the sectoral profit rates, and u_1 and u_{k1} are the rates of capacity utilization for sectors 1 and $k1$, respectively. α_1 and α_{k1} measure the influence of the investment to the profit rate for sectors 1 and $k1$, respectively, while β_1 and β_{k1} measure the sensibility of the growth rate of investment to the capacity utilization for sectors 1 and $k1$, respectively, and captures the accelerator effect: a high rate of capacity utilization induces firms to expand capacity in order to meet anticipated demand while low utilization induces firms to contract investment. The parameters $\theta_i, \alpha_i, \beta_i$ are all positive. After some algebraic manipulation, one shows that the overall growth rate of the capital stock in this economy, denoted by g, is given by:

$$g = \frac{I}{K} = k_1 g_1 + k_{k1} g_{k1} \tag{27}$$

By replacing expressions (25) and (26) into expression (27), such equation shows that we can obtain expression (14) from disaggregated investment functions, a point raised by Park. According to him: "[t]he above-mentioned 'reduced-form' equation must be obtained by 'reducing' sectoral investment functions to the aggregate counterpart." Let us assume the following saving function: $S = r_1 K_1 + r_{k1} K_{k1}$, which considers that workers do not save and the propensity of savings of capitalists is equal to one. By dividing the total savings by the stock of capital K we obtain:

$$\frac{S}{K} = r_1 k_1 + r_{k1} k_{k1} \tag{28}$$

Where $k_1 = \frac{K_1}{K}$ and $k_{k1} = \frac{K_{k1}}{K}$. It follows that:

$$k_1 + k_{k1} = 1 \tag{29}$$

In equilibrium, total savings are equal to total investment:

$$\frac{S}{K} = \frac{I}{K} \tag{30}$$

In what follows let us follow a slightly different approach to the Park model. From expressions (1), (2), (6) and (7), it is possible to obtain after some algebraic manipulation:

$$\frac{\tau_1}{1+\tau_1} = \frac{p_{k1}}{p_1}\frac{r_1 v_1}{u_1} \tag{31}$$

$$\frac{\tau_{k1}}{1+\tau_{k1}} = \frac{r_{k1} v_{k1}}{u_{k1}} \tag{32}$$

The sectoral profit-share for sectors 1 and $k1$ can be written respectively as: $\pi_1 = \frac{r_1 K_1 p_{k1}}{p_1 X_1}$ and $\pi_{k1} = \frac{r_{k1} K_{k1} p_{k1}}{p_{k1} X_{k1}}$. Then after some algebraic manipulation, the profit-share of sectors 1 and $k1$ may be written as:

$$\pi_1 = p\frac{v_1 r_1}{u_1} \tag{33}$$

$$\pi_{k1} = \frac{v_{k1} r_{k1}}{u_{k1}} \tag{34}$$

Where $p = \frac{p_{k1}}{p_1}$. Then it is possible to conclude that: $\pi_1 = \frac{\tau_1}{1+\tau_1}$ and $\pi_{k1} = \frac{\tau_{k1}}{1+\tau_{k1}}$. Within such framework, it is possible to conclude that balanced growth in the long run is more than a hypothesis as posed by expression (11). By considering the system formed by expressions (3), (5), (11), (12), (17), (18), (19), (20), (21), (22), (23), (24), (27) and (28), we have fourteen equations and fourteen unknowns, namely k_i, g_i, r_i, u_i, p_i, g, x, w and λ, which is perfectly determined, and has the following solution:

$$r_1^* = r_{k1}^* = \frac{\theta_{k1}\pi_{k1}}{(1-\alpha_{k1})\pi_{k1} - \beta_{k1}v_{k1}} \tag{35}$$

$$g^* = g_1^* = g_2^* = \frac{\theta_2\pi_{k1}}{(1-\alpha_{k1})\pi_{k1} - \beta_{k1}v_{k1}} \tag{36}$$

$$u_1^* = \frac{\left[\frac{\theta_{k1}\pi_{k1}}{\pi_{k1}(1-\alpha_{k1}) - \beta_{k1}v_{k1}}\right](1-\alpha_1) - \theta_1}{\beta_1} \tag{37}$$

$$u_{k1}^* = \frac{v_{k1}\theta_{k1}}{\pi_{k1}(1-\alpha_{k1}) - \beta_{k1}v_{k1}} \tag{38}$$

$$\lambda^* = \pi_{k1} \tag{39}$$

$$x^* = \frac{\lambda^*}{1-\lambda^*} = \frac{\pi_{k1}}{1-\pi_{k1}} \tag{40}$$

$$k_1^* = \pi_{k1} \tag{41}$$

$$k_2^* = (1 - \pi_{k1}) \tag{42}$$

$$w^* = \frac{1-\pi_{k1}}{(1-\pi_{k1})l_1 + \pi_{k1}l_{k1}} \tag{43}$$

$$p_1^* = \frac{W[(1-\pi_{k1})l_1 + \pi_{k1}l_{k1}]}{1-\pi_{k1}} \tag{44}$$

$$p_{k1}^* = \frac{W[(1-\pi_{k1})l_1 + \pi_{k1}l_{k1}]}{1-\pi_{k1}} \left\{ \frac{\pi_1\pi_{k1}[\theta_{k1}(1-\alpha_1) - \theta_1(1-\alpha_{k1})] + \theta_1\pi_1\beta_{k1}v_{k1}}{\theta_{k1}[\pi_{k1}(1-\alpha_{k1}) - \beta_{k1}v_{k1}]} \right\} \tag{45}$$

One of the important outcomes of our approach is that it is possible to determine the value of the rate of investment allocation, namely λ. Araujo and Teixeira (2002) have found the value of the rate of investment allocation consistent with the fulfilment of capital accumulation conditions. Here one of the outcomes of the analysis presented is that it allows us to determine the value of the rate of investment allocation consistent with the tendency of profit rate equalization, a criterion that was not taken into account for the above-mentioned authors. From expression (42) we conclude that such rate should equal the profit share of sector $k1$.

4. The Pasinetti Model and the Connection with the Pos-Kaleckian Framework

The Pasinettian model has a robust normative flavour insofar as it shows the requirements for an economic system to be in equilibrium and its difficulties, but it does not explain the actual position of the economic system. Besides, when moving from one sector to a multi-sectoral view of the growth process, it allows us to consider dimensions of the consumer choice that either one or two-sector model cannot take into account insofar as the only possibility of substitution occurs between current and future consumption in these models. Hence, when we move to a multi-sectoral approach, a fundamental change arises: workers may choose different patterns of consumption according to the evolution of their preferences.

Pasinetti (1981, 1993) seeks to understand these dynamics through a multisector model in which changes in macroeconomic magnitudes relates to changes in the economy composition, which are permanent and irreversible. Pasinetti's model has three fundamental elements: 1) pre-institutional feature; 2) natural economic system; 3) vertically integrated sectors [Garbelline and Wikierman (2014)]. The pre-institutional conception does not mean that it is a pre-industrial model; on the contrary, Pasinetti proposes to understand the nature of production process in any industrial system, that is why the model abstract from institutions. This fact demonstrates Pasinetti's focus on the primary and natural features[6] of the economic system [Garbelline and Wikierman (2014)].

As remarked, Pasinetti's goal is to develop a framework which focuses on the natural features of the economic system, consisting of (i) the commodities price structure, (ii) production structure and (iii) behavior of the wage rate and profit rate. In the exposition of these structures, Pasinetti emphasizes that learning constitutes the basic elements of the wealth of nations because knowledge affects both the production and consumption process. Given the pre-institutional nature and the natural features, Guarlezi (1996) observes that Pasinetti's multisector model come out as a system that deals with the requirements for full employment so that structural change emerges within the framework of equilibrium growth model. However, it is essential to qualify the concept of equilibrium growth in Pasinetti, because given structural changes the equilibrium does not represent the 'normal position' for which the economy tends in the long run.

The third fundamental feature of Pasinetti's model is that the economic sectors are vertically integrated. That device allows focussing on the final demand, with physical coefficients of labor and productive capacity all summarizing all previous phases of the production processes. It is important to emphasize that Pasinetti vertical integration is a way of representing reality and not describing it. Moreover, vertical integration does not consist of a second step in understanding interindustrial relations. Pasinetti (1990) observes that the concept of vertical integration is not sensitive to the technological inter-

[6] Pasinetti call natural features what the classics means as normal.

industrial variations. Given these fundamental features, we deal with the formal description of Pasinetti's structural change model (1981, 1993) and show that structural change is related to the evolution of technology and demand coefficients. Although Pasinetti recognizes technological progress as the *primum movens* of industrial society, this section presents the model without technological progress, because the objective is comparing and connecting it with Kaleckian model. The hypotheses adopted in the model are: all workers receive the same wage, and the n-th sector is the family sector. For each final consumption good, there is one capital good input, and there is no capital depreciation[7].

As highlighted, Pasinetti assumes the vertically integrated production process, in the sense that all inputs reduce to labor and capital inventories. Thus, in the productive system, there are two outflows: a) labor supply outflow of the n-th sector to the i-th sectors which produce capital goods and final consumer goods; b) outflow of goods from the i-th sectors to the n-th sector. That is, there are $n-1$ sectors that demand labour and supply products and the n-th sector that supplies labour and exerts a demand for the goods produced. Figure 1 below shows the outflows a) and b) of the model.

The model variables are:

- X_i: quantity of consumption good i;
- X_n: labor quantity;
- X_{in}: consumption demand of i-th sector by the n-th sector;
- X_{ni}: labor input to i sector;
- $a_{ni} = X_{ni}/X_i$: per capita production coefficient in consumption good sector;
- $a_{in} = X_{in}/X_n$: per capita demand coefficient in consumption good sector;

[7] This hypothesis is not adopted by Pasinetti (1981), however to facilitate the understanding of the model and to achieve the objective of this chapter we use it.

- p_i: final demand price in i sector;

- w: wage;

- X_{ki}: quantity of capital goods in i sector;

- X_{nki}: labor quantity used in the capital goods ki-th sector;

- X_{kin}: capital good demanded by the n-th sector;

- $a_{nki} = X_{nki}/X_{ki}$: per capita production coefficient in capital goothe d sector;

- $a_{kin} = X_{kin}/X_n$: per capita demand coefficient;

- r_i:i sector profit rate;

- p_{ki}: capital good price in i sector;

From the initial hypotheses and variables described above, Pasinetti shows the commodity production system and the commodity price model, both composed of a) series of $n-1$ capital goods inventories: $K_1, \ldots.,\ K_{n-1}$; b) a series of $2(n-1)$ technical coefficients: a_{n1}, a_{n2},......., a_{nn-1}, a_{nk} , a_{nk2}, ..., $a_{nk(n-1)}$; c) a series of n-1 final goods consumption coefficients: a_{1n}, a_{2n}, ..., $a_{(n-1)n}$; d) a series of n-1 capital goods consumption coefficients: a_{k1n}, a_{k2n}, ..., $a_{k(n-1)n}$. Assuming that $a_{kin} > 0$ and $a_{nki} > 0$, for i = 1, ..., n, the physical quantities system may be expressed by:

$$\begin{cases} X_i - a_{in}X_n = 0 \\ X_{ki} - a_{kin}X_n = 0 \\ X_n - \sum_{i=1}^{n-1} a_{ni}X_i - \sum_{i=1}^{n-1} a_{nki}X_{ki} = 0 \end{cases} \tag{46}$$

We can write the monetary system as:

$$\begin{cases} p_i - a_{ni}w - r_i p_{ki} = 0 \\ p_{ki} - a_{nki}w - r_i p_{ki} = 0 \\ w + \sum_{i=1}^{n-1} a_{in}\, p_{ki} r_i - \sum_{i=1}^{n-1} a_{in} p_i - \sum_{i=1}^{n-1} a_{kin} p_{ki} = 0 \end{cases} \tag{47}$$

In the Pasinetti model, the capital goods production depends only on labor, here (46) and (47) depends not only on labour but also on capital goods. That is assumed to maintain the similarity towards Kaleckian two sectors model without however undermining the essence of Pasinetti's model. In the same way, we adopt the convention that $l_i = a_{ni}$, $l_{ki} = a_{nki}$ and $L = X_n$. Pasinetti (1983) notes that both systems, (46) and (47), are linear and homogeneous. Thus, in order to have a non-trivial solution (quantities and prices equal to zero), it is necessary that the determinant of the coefficient matrix is equal to zero. The determinant for both systems is:

$$\sum_{i=1}^{n-1} a_{in}a_{ni} + \sum_{i=1}^{n-1} a_{kin}a_{nki} = 1 \tag{48}$$

Pasinetti (1981, 1993) refers to expression (48) as the full effective demand condition, which allows the economy to reproduce itself with full employment. The terms $a_{in}a_{ni}$ and $a_{kin}a_{nki}$ represent the relationship between the final product of each sector *i* and the consumption expenditures in this same sector. Thus, if (48) is satisfied, all produced goods are consumed, with full expendituthe re of income and full employment of the labor force. The macroeconomic equilibrium condition, if satisfied, yields the following solution for the physical quantities system (46):

$$\begin{cases} X_i = a_{in}X_n \\ X_{ki} = a_{kin}X_n \end{cases} \tag{49}$$

The solution for the monetary system, (47), is given by:

$$\begin{cases} p_i = \left(a_{ni} + \frac{r_i}{1+ r_i} a_{nki}\right) w \\ p_{ki} = \frac{1}{1+ r_i} a_{nki}w \end{cases} \tag{50}$$

Equations (49) show the physical quantities solution for each vertically integrated sector and show that goods production rely exclusively on demand since they are proportional to consumption coefficients. In turn, (50) shows the monetary system solution and show that prices are directly proportional to the amount of labor required for their production.

Therefore, as in the Kaleckian model, it is the demand that determines the amount produced in the economy. In this way, if $\sum_{i=1}^{n-1} a_{in}a_{ni} + \sum_{i=1}^{n-1} a_{kin}a_{nki} < 1$, in the dual system this means that: a) $\sum_{i=1}^{n-1} a_{ni}X_i < X_n$ that is, the total employment demanded is lower than the labor available in the economy, there is underemployment; b) $\sum_{i=1}^{n-1} a_{in}p_i < p_n$, that is, the average per capita expenditure is lower than the income received by the workers, there is underconsumption. On the other hand, if $\sum_{i=1}^{n-1} a_{in}a_{ni} + \sum_{i=1}^{n-1} a_{kin}a_{nki} > 1$, we conclude that: a) $\sum_{i=1}^{n-1} a_{ni}X_i > X_n$, that is, the employment demanded is higher than the available labor force; b) $\sum_{i=1}^{n-1} a_{in}p_i > p_n$, that is, the average per capita expenditure is higher than the income received by the workers, there is overconsumption.

Pasinetti when dealing with the macroeconomic equilibrium concept aimed to demonstrate how difficult it is to maintain full employment. To show this, he explains that even (48) is fulfilled in the first period of the analysis; it will not be satisfied later on. There is another condition that should be met to guarantee the equilibrium, which is a condition related to the sectoral capital accumulation. To derive it let us bear in mind that each sector has to be endowed with the necessary capital goods to produce the amount of consumption goods demand. Pasinetti (1981) has chosen a particular way of measuring the capital goods, which in equilibrium requires that the capital stock should be equal to the good consumer final produced, namely:

$$K_i = X_i \tag{51}$$

With this choice, Pasinetti intends to measure the capital goods by the number of consumption goods that they can produce. The difficulty of maintaining the equilibrium is perceived more efficiently by introducing time in the model that until now is static. Pasinetti (1981), considers two notions of time: a) time as a succession of finite periods, in such a way structural changes happens between periods; b) time is continuous, in this case, finite periods are infinitesimal. To keep the model simplified, Pasinetti adopts continuous time. Given that his objective is to discuss the economic variables evolution over time, the first additional hypothesis introduced by Pasinetti is the population variation (X_n), which consequently results in the variation of the effective demand (a_{in}), since it is assumed that consumers change their consumption patterns among sectors.

To keep the analysis consistent with the Kaleckian model, let us assume that there is no populational growth, and that total population is equal to the full labour force. There is no technological progress, so a_{ni} and a_{nki} are constant; The comsuption pattern changes at a rate φ_i for each of the i-th sectors. Analytically, these hypotheses are represented by:

$$X_n(t) = X_n(0) \tag{52}$$

$$a_{in}(t) = a_{in}(0)e^{\varphi_i t} \tag{53}$$

$$a_{ni}(t) = a_{ni}(0) \tag{54}$$

$$a_{nki}(t) = a_{nki}(0) \tag{55}$$

Equation (52) shows that population is constant over time. Expression (53) shows that the final consumption goods and capital goods coefficients vary for each sector at a rate φ_i . Expressions (54) and (55) show that the technical (production) coefficients in the final goods sector and in the capital goods sector are constant. From expression (51), it possible to show that the dynamic sectoral equilibrium in terms of capital accumulation requires that:

$$\dot{K}_i = \dot{X}_i, \text{for } i = 1, 2, \ldots., n-1 \tag{56}$$

From the first line of (49) and (53), we conclude that the dynamic path of production of the i-th sector is given by: $X_i(t) = a_{in}(0)X_n(0)e^{\varphi_i t}$. By taking logs and the time derivative, we conclude after some algebraic manipulation that the growth rate of the consumption goods sector is given by: $\frac{\dot{X}_i}{X_i} = \varphi_i$. By substituting this result into (56), it yields after some algebraic manipulation the following relation between the sectoral coefficients of investment and demand for i-th good[8]:

$$a_{kin(t)} = \varphi_i \, a_{in(t)} \tag{57}$$

Pasinetti (1981) refers to expression (58) as a capital accumulation condition, which should be fulfilled to endow every consumption goods sector with the capital goods required to meet the demand requirements. From the second line of expression (49), it is then possible to show that the growth rate of the capital goods sector, in the long run, is also given by $\frac{\dot{X}_{ki}}{X_{ki}} = \varphi_i$. Then by considering that g_i and g_{ki} stand for the growth rate of sectors i and ki respectively, we conclude that in the long run:

$$g_i = g_{ki} = \varphi_i \tag{58}$$

Pasinetti (1981) also shows that there exists a sectoral rate of profit that is compatible with expression (58). He refers to it as the natural rate of profit, and according to him, the proportionality between the rate of profit to the sectoral rate of growth emerges as a natural requirement to endow the economic system with the necessary productive capacity to fulfil the expansion of demand, which is given by:

$$r_i = \varphi_i \tag{59}$$

The natural rate of profit corresponds to the rate of profit required for the equilibrium expansion of productive capacity in each vertically integrated sector to take place. By considering the concept of natural rate of profit, as advanced by Pasinetti allows us to consider a variable which provides the means to promote capital accumulation in equilibrium. Note that if $r_i < \varphi_i$ then capitalists in the i-th sector will not have the necessary amount of resources to invest in such sector in order to meet the expansion of

[8] See Pasinetti (1981, p. ?) for the derivation of this result.

demand. If $r_i > \varphi_i$ then capitalist will overinvest in the i-th sector leading to excess of productive capacity.

From expressions (58) and (59), we conclude that in the long run $r_i = g_i$, a result akin to expression (15). Although we do not make explicit considerations with respect to the evolution of demand in the pos-Kaleckian model, it becomes clear that if the sectoral growth rate of demand is equal to φ_i, then the sectoral profit rate given by expression (15) is the one required to endow the capitalist class with the required funds to reinvest fulfilling the expansion of demand in a specific vertically integrated sector. This fact shows that within a multi-sectoral growing system, each vertically integrated sector has its own natural profit rate insofar as the growth rate of capital accumulation should be sensitive to the evolution of demand in that sector.

A possible interpretation of the Pasinettian concept of the 'natural rate of profit' is that the it is a warranted rate of profit, which when adopted allows endowing the sector with the units of productive capacity necessary to fulfil demand requirements. Hence, as point out by Pasinetti (1981, p. 130), "there are as many natural rates of profit as there are rates of expansion of demand (and production) of the various consumption goods." Within this setup, there will be profit equalization just within the vertically integrated sector, with the consumption and correspondent capital goods sector having the same profit rate. But in general, for sectors i and j, with particular growth rates of demand, we would expect different profit rates even in the long run.

From the above equations, it is possible to move from one model to another. However, a 'bridge' is required, which is represented by the Feldman model (1928). From the equilibrium solutions, it is possible to relate $p_{ki} = \frac{1}{1-r_{ki}} a_{nki} W$, of the Pasinetti system, with the mark-up pricing of the pos-Kaleckian model, namely $p_{ki} = (1 + \theta_{ki})Wl_{ki}$. By equalizain these expressions, one obtains:

$$r_{ki} = \frac{\theta_{ki}}{1+\theta_{ki}} \tag{61}$$

Expression (61) shows us the relation between Kalecki mark-up and Pasinetti sector profit rate. Carrying out the same exercise for the first sector, one has:

$$p_i = [\,1 + \frac{r_i}{1-r_{ki}}\frac{a_{nki}}{a_{ni}}]a_{ni}W \tag{62}$$

Thus, under the Pasinetti pre-institutional model it is possible to obtain that Kalecki mark-up for the first sector is:

$$\frac{r_i}{1-r_{ki}}\frac{a_{nki}}{a_{ni}} = \theta_i \tag{63}$$

Substituting (61) into (63), it yields:

$$r_i = \frac{\theta_i}{1+\theta_{ki}}\frac{a_{ni}}{a_{nki}} \tag{64}$$

Given the above results, it is possible to conclude that from Pasinetti model one obtains the Kalecki conclusions. Note that (61) and (64) are nothing but expressions (16) and (17) when we assume, according to Pasinetti, full productive capacity utilization, $\frac{u_i}{v_i} = \frac{u_{ki}}{v_{ki}} = 1$. The deduction of the results of the Kaleckian model from Pasinetti model can also be obtained from the viewpoint of the profit share. See initially the profit share of the capital goods sector: $\pi_{ki} = r_{ki}v_{ki}u_{ki}^{-1}$, which implies that:

$$r_{ki} = \frac{\pi_{ki}u_{ki}}{v_{ki}} \tag{65}$$

Equation (65) is nothing but the Kaleckian profit-cost curve (16) for the capital goods sector. For the consumer goods sector: $\pi_i = \frac{p_{ki}}{p_i}r_i v_i u_i^{-1}$, which implies that:

$$r_i = (\frac{p_{ki}}{p_i})\,\frac{\pi_i u_i}{v_i} \tag{66}$$

Equation (65) is not precisely the Kaleckian profit-cost curve (17), due to $\frac{p_{ki}}{p_i}$ ratio. This ratio exists because in the first sector the production input is not equal to the product as in the second sector, so there are relative prices. However, if we replace (6), (7) and (65) in (66) we, find $\frac{u_i}{v_i} = 1$. That is, (65) is the profit-cost curve (17) for the consumption goods sector if the system is in full productive capacity. To support the viewpoint that the Pasinetti model contains in itself all the equations of Kalecki, it remains to be derived the equations (3) and (4). Equation (3) is easily identified if we adopt the Pasinettian nomenclature, $1 = W\left(a_{n1} + a_{nk1}\frac{X_{k1}}{X_1}\right)$, which allows us to conclude that:

$$X_i = W(a_{ni}X_i + a_{nki}X_{ki}) \tag{67}$$

Expression (66) shows that the total quantity of consumer goods produced is equal to labor employed in sectors i and ki. Finally, to discuss (4), which shows that the quantity of capital goods produced is equal to the sum of investment in both sectors, it is necessary to bear in mind that a fraction of the capital goods sector is allocated to itself, while the remaining is allocated to the consumption goods sector. This view of capital accumulation was further developed in the context of the pos-Kaleckian model in the previous section, and rests on the view advanced by Feldman (1928). These authors have adopted the important assumption that, once installed, capital is not reused in another sector (non-shifitability assumption).

We see that practically all the hypotheses adopted by Feldman are in line with the pos-Kaleckian model, which means that we can build also built links with the Pasinettian model. In fact, the task was accomplished by Araujo and Teixeira (2002) who have shown that the Feldman model may is a particular case of the Pasinetti model. That is, if there is no proper long-run production of capital goods, the economy will not be able to keep the stock balance proposed by Pasinetti. Thus, from (21), (22), (56) and (4.22) we obtain for each sector the following results:

$$X_i \varphi_i = (1 - \lambda_i) X_{ki} \tag{68}$$

$$X_{ki} \varphi_i = \lambda_i X_{ki} \tag{69}$$

We know from expression (58) that g_i of Kalecki model. In this way, one can rewrite (68) and (69) as:

$$X_i g_i = (1 - \lambda_i) X_{ki} \tag{70}$$

$$X_{ki} g_{ki} = \lambda_i X_{ki} \tag{71}$$

Adding (70) to (71), one obtains:

$$x_i = g_i + g_{ki} x_i \tag{72}$$

Where x_i stands for the output of the investment good relative to the output of the consumption good for the i-th vertically integrated sector. Equation (72) is nothing but equation (4) of the Kaleckian two-sector model when $\frac{v_i}{u_i} = \frac{v_{ki}}{u_{ki}} = 1$, which as stated above is the underlying condition for stocks equilibrium in Pasinetti model. From the results, it is shown that all equations of Kaleckian two sector model are obtained from Pasinetti two sector model under the hypothesis of $K_i = X_i$. This happens, as pointed out in the previous chapter, because the objective of Pasinetti was to obtain the conditions for the maintenance of long run equilibrium growth. As seen in Pasinetti, capital goods are sector specific and the displacement between sectors is not assumed, but it is shown at which rate the capital good must grow so that the sector remains in long run equilibrium. That is, Pasinetti does not adopt the displacement of capital goods between the sectors, but only intra sectors, there is displacement of capital goods from his industry to the industry of the consumption goods. The same is assumed in the Kaleckian two sector model.

5. Concluding Remarks

The present article shows that the two-sector Kaleckian model may be view as a particular case of the multi-sectoral Pasinettian model of structural change if we introduce mark-up pricing and the possibility of under-capacity utilization in the Pasinetti framework. The standpoint of the analysis is the concept of vertical integration which allows us to establish a correspondence between the two approaches. With such method, it was possible to show the actual structural dynamics also depends on the distributive features of the economy and not only on the evolution patterns of demand and technological progress as in the Pasinettian view. In particular, we confirmed the results by Araujo and Teixeira (2011, 2012), who showed that each sector has a specific regime of growth. Besides, from the Pasinettian insight that each sector has a natural rate of profit, which is consistent with the fulfillment of the demand requirements, it is possible to conclude that the equalization of profit rates occurs just within the vertically integrated sectors, but not across sectors. The present paper contributes to conceptualizing growth based on the principle of effective demand, in which Kaleckian and Pasinettian frameworks are shown to be consistent.

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